

1. In the SCHEDULING ON UNRELATED PARALLEL MACHINES problem, we are given  $n$  jobs and  $m$  machines. We want to assign every job to some machine. The time of running job  $j$  on machine  $i$  is given by  $p_{i,j}$ . The goal is to minimize the *maximum total processing time* required by a machine.

Show that this problem cannot be  $(3/2 - \varepsilon)$ -approximated.

**Hint.** Reduce from 3-DIMENSIONAL MATCHING, which is an NP-hard problem. In this problem, we are given sets  $A, B, C \subseteq [n]$  and a family of sets  $\mathcal{F} \subseteq A \times B \times C$ . The goal is to find a subset  $\mathcal{F}' \subseteq \mathcal{F}$  such that every element of  $A$ ,  $B$ , and  $C$  is contained in *exactly* one of the sets of  $\mathcal{F}'$ .

2. Show that INDEPENDENT SET cannot be  $(7/8 + \varepsilon)$  approximated.

**Hint.** Standard reduction from 3SAT suffices. What kind of  $L$ -reduction is this?

3. The Cook-Levin theorem states that 3SAT is NP-hard. We want to show that its standard proof does not show inapproximability of MAX3SAT.

The proof defines for every NP language a reduction  $f$  such that

$$x \in L \Leftrightarrow f(x) \in \text{3SAT}.$$

Prove that there exists  $x \notin L$  such that  $f(x)$  is a formula with  $m$  clauses and a assignment satisfying more than  $(1 - o(1))m$  of them.

**Hint.** What does the reduction *encode*? Do not worry about specific numbers, this is enough to get the idea.